Zero Range Distorted Wave Born Approximation

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1. General Description of DWUCK4

The computer code DWUCK4 calculates the scattering and reaction observables for binary nuclear reactions using the distorted wave Born approximation (DWBA). The calculations are performed using a zero-range interaction. This restriction is, in general, not important for inelastic scattering, however it is an approximation for the calculation of particle transfer reactions. The spin of the projectiles may be any combination of spin 0, spin 1/2 or spin 1. The following physics description uses the notation and formulation of ref. [1] which describes nuclear reaction theory in detail.

The computer code calculates a transition amplitude for the reaction A(a,b)B of the form

$$T = J \int d^3r_b \int d^3r_a \chi^{(-)}(\vec{\mathbf{k}}_\mathbf{f}, \vec{\mathbf{r}}_\mathbf{b})^* \, \langle bB|V|aA\rangle \, \chi^{(+)}(\vec{\mathbf{k}}_\mathbf{i}, \vec{\mathbf{r}}_\mathbf{a}), \tag{1.1}$$

where $\chi^{(-)}$ and $\chi^{(+)}$ are the distorted waves, $\vec{\mathbf{r}}_\mathbf{a}$ and $\vec{\mathbf{r}}_\mathbf{b}$ are the relative coordinates for the systems (a,A) and (b,B), respectively, and J is the Jacobian for the transformation to these coordinates. The quantity $\langle bB|V|aA\rangle$ is the form factor for the reaction and must contain a delta-function between the coordinates $\vec{\mathbf{r}}_\mathbf{a}$ and $\vec{\mathbf{r}}_\mathbf{b}$. The distorted waves $\chi^{(\pm)}(\vec{\mathbf{k}}, \vec{\mathbf{r}})$ asymptotically describe a plane wave of momentum $\vec{\mathbf{k}}$ plus an outgoing (or incoming) spherical scattered wave which in the case of no Coulomb potential has the form,

$$\chi^{(\pm)}(\vec{\mathbf{k}}, \vec{\mathbf{r}}) \longrightarrow e^{i\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} + f(\theta)\frac{e^{\pm ikr}}{r}. \tag{1.2}$$

The final distorted wave which has an incoming scattered wave condition is related to the solution with the outgoing waves by

$$\chi^{(-)*}(\vec{\mathbf{k}}, \vec{\mathbf{r}}) = \chi^{(+)}(-\vec{\mathbf{k}}, \vec{\mathbf{r}}). \tag{1.3}$$

When the initial and final projectiles have spin the distorted waves are matrices in spin space,

$$\chi^{(\pm)}(\vec{\mathbf{k}}, \vec{\mathbf{r}})\eta_{s,m} = \sum_{m'} \chi^{(\pm)}_{m,m'}(\vec{\mathbf{k}}, \vec{\mathbf{r}})\eta_{s,m'}, \tag{1.4}$$

where the $\eta_{s,m}$ are spin functions,

$$\chi^{(\pm)}_{m,m'}(\vec{\mathbf{k}}, \vec{\mathbf{r}}) = \frac{\sqrt{4\pi}}{kr} \sum_{J,L} i^L \sqrt{2L+1}\, \chi_{JLs}(k,r)\, (L\, s\, M\, m|J\, M')\, (L\, s\, M' - m'\, m'|J\, M')\, Y_L^{M'-m'}(\hat{\mathbf{r}}) d^L_{0,M'-m'}(\hat{\mathbf{k}}), \tag{1.5}$$

and the $d^L_{0,m}$ are the rotation functions for integer spin [2]. The radial part of the distorted waves satisfies the equation,

$$\Big(\frac{d^2}{dr^2} + k^2 - \frac{L(L+1)}{r^2} - \frac{2\mu}{\hbar^2}[U(r) + U_c(r) + U_{Ls}(r)\mathbf{L}.\mathbf{s}]\Big)\chi_{JLs}(k,r) = 0. \tag{1.6}$$

Here U is a central potential with real and imaginary parts, U_c is a Coulomb potential for a uniform charge distribution of radius R_c and U_{Ls} is a spin-orbit potential. The radial functions $\chi_{JLs}(k,r)$ satisfy the boundary conditions $\chi_{JLs}(k,0)=0$ at the origin and

$$\chi_{JLs}(k,r) \longrightarrow \frac{i}{2}[H_L^-(kr) - \eta_L^J H_L^+(kr)]\, e^{i\sigma_L}. \tag{1.7}$$

for large r (where U and U_{Ls} can be neglected). Here $H_L^{\pm}(kr) = G_L(kr) \pm iF_L(kr)$ are the outgoing (+) and the incoming (−) Coulomb waves, η_L^J is the elastic scattering S matrix and σ_L is the Coulomb phase shift.

The form factor $\langle bB|V|aA\rangle$ contains the nuclear structure information and is expressed by

$$\begin{aligned}\langle J_B M_B s_b m_b|V|J_A M_A s_a m_a\rangle = \sum_{lsj} & B_{lsj}(J_A\, j\, M_A\, M_B - M_A|J_B\, M_B)(s_a\, s\, m_b\, m_a - m_b|s_a\, m_a) \\ & \times(l\, s\, m\, m_a - m_b|j\, M_A - M_B) f_{lsj}(r_c)\delta(r_b - \frac{A}{B}r_a) i^{-l} Y_l^m(\hat{r}_a)^*.\end{aligned} \tag{1.8}$$

The quantity B_{lsj} is a measure of the strength of the interaction and is equal to $\sqrt{(2s+1)/(2s_a+1)}A_{lsj}$ of ref. [1]. The function f_{lsj} is the radial form factor for the reaction. The details of the reaction model are contained in f_{lsj} and are discussed in section 2 for typical cases. In the above equation, the angular momenta must satisfy the triangular relations,

$$\vec{\mathbf{j}} = \vec{\mathbf{J}}_{\mathbf{B}} - \vec{\mathbf{J}}_{\mathbf{A}}, \qquad \vec{\mathbf{s}} = \vec{\mathbf{s}}_{\mathbf{a}} - \vec{\mathbf{s}}_{\mathbf{b}}, \qquad \vec{\mathbf{l}} = \vec{\mathbf{j}} - \vec{\mathbf{s}}.$$

With the definitions of $\langle bB|V|aA\rangle$ and the distorted wave functions $\chi^{(\pm)}_{m,m'}$, we can write down the transition amplitude,

$$T^{M_A M_B; m_a m_b} = \frac{\sqrt{4\pi}}{k_a k_b} \sum_{l,s,j} \sqrt{2l+1}\, B_{lsj}(J_A\, j\, M_A\, M_B - M_A|J_B\, M_B) S_{lsj}^{m m_a m_b}, \tag{1.9}$$

where the angle-dependent amplitude S is written as

$$S_{lsj}^{m m_a m_b} = \sum_{L_b} \beta_{lsj;L_b}^{m m_a m_b}\, P_{L_b}^{m_a - m - m_b}. \tag{1.10}$$

The inelastic scattering amplitudes β are given by

$$\begin{aligned}\beta_{lsj;L_b}^{m m_a m_b} = & \sum_{J_a L_a J_b m} (L_a\, s_a\, 0\, m_a|J_a\, m_a)(L_b\, s_b\, m_a - m - m_b\, m_b|J_b\, m_a - m) \\ & \times(J_b\, j\, m_a - m\, m|J_a\, m_a)(2L_b+1)(L_b\, l\, 0\, 0|L_a\, 0) \\ & \times\sqrt{(2s_a+1)(2j+1)(2J_b+1)(2L_a+1)} \begin{Bmatrix} L_b & s_b & J_b \\ l & s & j \\ L_a & s_a & J_a \end{Bmatrix} \\ & \times I_{J_a L_a J_b L_b}^{lsj}\, i^{L_a - L_b + l}.\end{aligned} \tag{1.11}$$

In the above expression the usual nine-j symbol, {} [2], appears. The radial integrals I are

$$I_{J_a l_a J_b l_b}^{lsj} = \frac{CB}{A^2} \int_0^\infty dr_c\, \chi^{(-)}_{J_b L_b}(k_b, \frac{A}{B}r_a)\, f_{lsj}(r_c)\, \chi^{(+)}_{J_a L_a}(k_a, r_a), \tag{1.12}$$

where the mass of the form factor core is C. The radial integrals I and the scattering amplitudes β may be printed in the output (see section 3 below).

The differential cross section for the reaction A(a,b)B may now be expressed in terms of the transition amplitude T,

$$\begin{aligned}\frac{d\sigma(\theta)}{d\Omega} &= \left(\frac{\mu_b}{2\pi\hbar^2}\right)^2 \frac{v_b}{v_a}\frac{1}{(2J_A+1)(2s_a-1)} \sum_{M_A M_B m_a m_b} |T^{M_A M_B; m_a m_b}|^2 \\ &= \frac{1}{4\pi}\frac{2J_B+1}{2J_A+1}\frac{1}{E_a E_b}\frac{k_b}{k_a}\frac{1}{2s_a+1} \sum_{m_a m_b m} |\sum_{lsj} \sqrt{2l+1} B_{lsj} S_{lsj}^{m m_a m_b}|^2,\end{aligned} \tag{1.13}$$

where E_a and E_b are the center of mass energies for the entrance and exit channels, respectively.

The computer program DWUCK4 computes the cross section with different normalizations depending upon whether the case being run is an inelastic scattering or a particle transfer reaction. In the case of inelastic excitations (where the mass of a and b are equal) we define

$$\sigma_{DW}^{lsj}(\theta) = \frac{1}{4\pi}\frac{1}{E_a E_b}\frac{k_b}{k_a}\frac{1}{2s_a+1} \sum_{m_a m_b m} |\sum_{lsj} S_{lsj}^{m m_a m_b}|^2. \tag{1.14}$$

For the case of particle transfer (where the masses a and b are unequal) we define

$$\sigma_{DW}^{lsj}(\theta) = \frac{1}{4\pi}\frac{1}{E_a E_b}\frac{k_b}{k_a}\frac{10^4}{2s_a+1} \sum_{m_a m_b m} |\sum_{lsj} \sqrt{2l+1} S_{lsj}^{m m_a m_b}|^2. \tag{1.15}$$

These choices are convenient for (p,p′) and (d,p) reactions respectively (see section 2). The cross sections are computed in units of fm^2/steradian where 1 fm^2 = 10^{-26} cm^2.

The physical cross sections which include reaction strengths and sums over the spin indices are

$$\frac{d\sigma^{lsj}(\theta)}{d\Omega} = \frac{2J_B+1}{2J_A+1}\frac{2l+1}{2j+1}|B_{lsj}|^2 \sigma_{DW}^{lsj}(\theta) \qquad \text{for equal masses,} \tag{1.16}$$

and

$$\frac{d\sigma^{lsj}(\theta)}{d\Omega} = \frac{2J_B+1}{2J_A+1}\frac{1}{2j+1}\frac{|B_{lsj}|^2}{10^4} \sigma_{DW}^{lsj}(\theta) \qquad \text{for unequal masses.} \tag{1.17}$$

In addition to the cross sections, DWUCK4 calculates certain spin observables. If we define a general spin operator $\sigma_{m_a m'_a; m_b m'_b}$, then the program will also calculate the expectation value of the spin operator by the following expression,

$$\langle\sigma\rangle = \frac{\sum_{m m_a m_b} \sum_{m'_a m'_b} S_{lsj}^{m m_a m_b *} (\sigma)_{m_a m_b; m'_a m'_b} S_{lsj}^{m m'_a m'_b}}{\sum_{m m_a m_b} |S_{lsj}^{m m_a m_b}|^2}. \tag{1.18}$$

The spin operator for the polarization of the final state projectile is

$$(\sigma_y)_{m_a m'_a; m_b m'_b} = (\sigma_y)_{m_b m'_b} \delta_{m_a, m'_a} \qquad \text{for spin } \tfrac{1}{2}$$

and

$$(\sigma_y)_{m_a m'_a; m_b m'_b} = (S_y)_{m_b m'_b} \delta_{m_a, m'_a} \qquad \text{for spin 1.}$$

Similarly, the spin operator for the polarization of the initial state projectile is

$$(\sigma_y)_{m_a m'_a; m_b m'_b} = (\sigma_y)_{m_a m'_a} \delta_{m_b, m'_b} \qquad \text{for spin } \tfrac{1}{2}$$

and

$$(\sigma_y)_{m_a m'_a; m_b m'_b} = (S_y)_{m_a m'_a} \delta_{m_b, m'_b} \qquad \text{for spin 1.}$$

In the case of the initial state the polarization is also the asymmetry observable.

For the case of spin-one particles, tensor analyzing powers are also calculated. The operators for these quantities are defined elsewhere [4] and will not be given here.

2. Specific Cases of Reactions.

a) Inelastic excitation using a collective nuclear model.

To first order in the deformation β_l the interaction V can be written as,

$$V(r,\theta) = -\beta_l \frac{R_0}{a} \frac{dV(x)}{dx} Y_l^0(\theta), \tag{2.1}$$

where $x = (r - R_0)/a$, $V(x)$ is the projectile-nucleus potential, and θ is the angle between the radius vector of the projectile and the nuclear symmetry axis. If we take the strong coupling form for the initial and final wave functions for the nucleus [3],

$$\psi^j_{JMK} = \sqrt{\frac{2J+1}{16\pi^2}} \left[\phi^j_K {D^J_{K,M}}^*(\alpha,\beta,\gamma) + (-1)^{J-j} \phi^j_{-K} {D^J_{-K,M}}^*(\alpha,\beta,\gamma)\right], \tag{2.2}$$

we can write

$$\begin{aligned} \left\langle J_B M_B s_b m_b | V | J_A M_A s_a m_a \right\rangle = -\beta_l \frac{R_0}{a} \frac{dV(x)}{dx} \sqrt{\frac{2J_A+1}{2J_B+1}} \\ \times (J_A\, l\, K\, 0 | J_B\, K)(J_A\, l\, M_A\, M_B - M_A | J_B\, M_B) Y_l^m(\hat{\mathbf{r}})^*. \end{aligned} \tag{2.3}$$

This form of the interaction gives the strength as,

$$B_{l0l} = \beta_l \sqrt{\frac{2J_A+1}{2J_B+1}} (J_A\, l\, K\, 0 | J_B\, K), \tag{2.4}$$

and the resulting cross section is,

$$\frac{d\sigma^{lsj}(\theta)}{d\Omega} = \beta_l^2 (J_A\, l\, K\, 0 | J_B\, K)^2 \sigma^{lol}_{DW}(\theta). \tag{2.5}$$

In many cases the inelastic scattering cross section is strongly affected by the Coulomb portion of the interaction between the projectile and target nucleus. This effect may be incorporated by a non-zero value of the parameter COUEX in input block 4 (see section 3 below). An additional contribution is added to the form factor during computation of the radial integrals of the form

$$\begin{aligned} f^{\text{coulomb}}_{l0l}(r) &= \text{COUEX} \frac{3\,\text{Z}\,\text{ZT} * e^2}{2l+1} \frac{R^l_{coul}}{r^{l+1}} \quad \text{for } r > R_{coul} \\ &= 0 \quad \text{for } r < R_{coul}\ , \end{aligned}$$

where the parameters Z, ZT, and $R_{coul} = r_{0c}\,\text{MT}^{\frac{1}{3}}$ are the charges and charge radius from the kinematic input line in input block 7. In this way the radius of the charge distribution for the form factor can be different from the charge radius used for the distorted waves. A value of COUEX = 1.0 will normalize the Coulomb amplitude in the conventional manner [1]. In order to calculate the integrals accurately to large radii, the program automatically uses an integration along an imaginary radius line beginning at the maximum radius specified in the input.

b) The (d,p) stripping reaction.

The matrix element for a stripping reaction A(a,b)B where a = b+x and B = A + x may be written as,

$$\begin{aligned}\left\langle J_B M_B s_b m_b|V|J_A M_A s_a m_a\right\rangle = \sum_{jl} S_{jl}^{\frac{1}{2}} R_{jl}(r_{xA})(l\, s\, m\, \mu - m|j\, \mu)(s_b\, s\, m_b\, m_a - m_b|s_a\, m_a) \\ \times(J_A\, j\, M_A\, M_B - M_A|J_B\, M_B) D(\mathbf{r_{xb}}) Y_l^{m\,*}(\hat{\mathbf{r}}_{\mathbf{xA}})\end{aligned} \tag{2.6}$$

where $S_{jl}^{\frac{1}{2}}$ is the spectroscopic amplitude, $R_{jl}(r_{xA})$ is the radial wave function for the transferred particle x in the target nucleus B and $D(\mathbf{r_{xb}})$ is the product of the projectile internal function times the interaction potential between the projectile components x and b. In order to evaluate the DWBA matrix element one makes use of the zero-range approximation,

$$D(\vec{\mathbf{r}}_{\mathbf{xb}}) = D_0 \delta(\vec{\mathbf{r}}_{\mathbf{x}} - \vec{\mathbf{r}}_{\mathbf{b}}). \tag{2.7}$$

The reaction strength factor B_{lsj} becomes for this case,

$$B_{lsj} = S_{jl}^{\frac{1}{2}}\, D_0, \tag{2.8}$$

where $S_{jl}^{\frac{1}{2}}$ is the spectroscopic amplitude. If the radial form factor in DWUCK4 is

$$f_{lsj} = R_{jl}(r_{xA}), \tag{2.9}$$

then the resulting cross section is written as

$$\frac{d\sigma^{lsj}(\theta)}{d\Omega} = \frac{2J_B+1}{2J_A+1}\frac{S_{lsj}}{2j+1}\frac{D_0^2}{10^4}\sigma_{DW}^{lsj}(\theta). \tag{2.10}$$

Typical deuteron models [1] give a value $D_0^2 \approx 1.5 \times 10^4$ MeV-fm^3.

A first order correction to the zero range approximation for single particle transfer reactions may be made by multiplying the form factor f_{lsj} by the function

$$\begin{aligned} W_{FR}(r) =& [1 + A(r)]^{-1} \qquad \text{Hulthén form} \\ =& \exp[-A(r)] \qquad \text{Gaussian form}\end{aligned}$$

where

$$A(r) = \frac{2}{\hbar^2}\frac{m_b m_x}{m_a} R^2 [E_b - V_b(r_b) + E_x - V_x(r_x) - E_a + V_a(r_a)].$$

Here R is the finite range parameter and E_a, E_b, E_x and V_a, V_b, V_x are the energies and potentials with respect to the target core for the light particles a, b and x. A positive value of the finite range parameter FNRNG in input block 4 will select the Hulthén form while a negative value will select the Gaussian form. Again typical models for the (d,p) reaction [1] give R $\approx$ 0.70 fm.

A non-local correction factor may also be needed by the use of a local equivalent potential for the distorted waves and for the transfered particle bound state. This factor again multiplies the form factor and for each particle is given by

$$W_{NL}(r_i) = \exp[\frac{\beta_i^2}{8}\frac{2m_i}{\hbar^2} V_i(r_i)]$$

where β_i is the non-locality parameter for particle i and is input on the kinematics input line as PNLOC in input blocks 5, 6 and 7.

In addition the program will properly compute the (d,p) crossection when the final nuclear state is unbound. The technique used is due to Vincent and Fortune [5]. This option is entered automatically when a positive energy is detected for the binding of the transfered particle to the target.

c) The (p,d) pickup reaction.

The pickup reaction is the inverse of the stripping reaction described in the preceding section. Hence we may write the cross section for the (p,d) reaction in terms of the (d,p) reaction cross section, using detailed balance, as

$$\begin{aligned}\frac{d\sigma_{pd}^{lsj}(\theta)}{d\Omega} &= \frac{2J_A+1}{2J_B+1}\frac{2s_a+1}{2s_b+1}\frac{k_a^2}{k_b^2}\frac{d\sigma_{dp}^{lsj}(\theta)}{d\Omega} \\ &= \frac{3}{2}\frac{S_{lsj}}{2j+1}\frac{D_0^2}{10^4}\sigma_{DW}^{lsj}(\theta),\end{aligned} \tag{2.11}$$

where σ_{DW}^{lsj} is the output from DWUCK4 with protons in the incident channel and deuterons in the exit channel.

d) Microscopic Interaction Model for Inelastic Scattering.

1) Central Interactions

This model considers the potential between the projectile and <u>one</u> of the nucleons in the target nucleus. It is convenient to expand this interaction in a Legendre polynomial series,

$$\begin{aligned}V(\vec{\mathbf{r}}-\vec{\mathbf{r}}_{\mathbf{1}}) &= V_0\sum_l (2l+1)v_l(r,r_1)P_l(\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}_{\mathbf{1}}) \\ &= 4\pi V_0\sum_l v_l(r,r_1)\sum_m Y_l^m(\hat{\mathbf{r}})Y_l^{m*}(\hat{\mathbf{r}}_{\mathbf{1}}).\end{aligned} \tag{2.12}$$

Using this form of the interaction we find for the matrix element of V,

$$\begin{aligned}\big\langle J_B M_B s_b m_b|V(\vec{\mathbf{r}}-\vec{\mathbf{r}}_{\mathbf{1}})|J_A M_A s_a m_a\big\rangle = \\ 4\pi V_0\sum_l (J_A\, l\, M_A\, M_B - M_A|J_B\, M_B)\big\langle J_B||v_l(r,r_1)i^l Y_l(\hat{\mathbf{r}}_{\mathbf{1}})||J_A\big\rangle \\ \times(s_b\, s\, m_b\, m_a - m_b|s_a\, m_a)(-i)^l Y_l^m(\hat{\mathbf{r}})^*\frac{1}{\sqrt{2J_B+1}}.\end{aligned} \tag{2.13}$$

The program DWUCK4 calculates the following expression for the radial factor [see eq (1.8],

$$f_l(r) = 4\pi V_0\sqrt{2J_A+1}\big\langle j_1' j_2 J_B||Y_l(\hat{\mathbf{r}}_{\mathbf{1}})||j_1 j_2 J_A\big\rangle\int R_{l_1' j_1'}(r_1)v_l(r,r_0)R_{l_1 j_1}(r_1)r_1^2 dr_1, \tag{2.14}$$

where the $R_{lj}(r_1)$ are normalized radial functions for the initial and final states of the nucleon in the target with angular momentum l and total angular momentum j and $\big\langle j_1' j_2 J_B||Y_l(\hat{\mathbf{r}}_{\mathbf{1}})||j_1 j_2 J_A\big\rangle$ is the reduced matrix element as defined by Edmonds [2]. The angular momentum j_2 is the coupled angular momentum of the $A-1$ spectator nucleons. The program prints out [RME in the output] the quantity, $\sqrt{4\pi}\big\langle j_1' j_2 J_B||Y_l(\hat{\mathbf{r}}_{\mathbf{1}})||j_1 j_2 J_A\big\rangle$.

If there is more than one particle in each configuration then the form factor must be scaled by the amplitude of that configuration and by the counting factor which takes into account the number of identical particles in the configuration. Thus, the strength of the reaction B_{l0l} is

$$B_{l0l}(j_1' j_2 J_B; j_1 j_2 J_A) = a_{j_1' j_2 J_B}a_{j_1 j_2 J_B}\sum_{i=1}^{n}\big\langle j_1' j_2 J_B||Y_l(\hat{\mathbf{r}}_{\mathbf{i}})||j_1 j_2 J_A\big\rangle\big\langle j_1' j_2 J_B||Y_l(\hat{\mathbf{r}}_{\mathbf{1}})||j_1 j_2 J_A\big\rangle. \tag{2.15}$$

Here, $a_{jj'J}$ are the amplitudes of the initial and final configurations which involve the factor for the n identical particles in the shell. If the strength of each configuration is scaled by B_{l0l}, then the cross section is

$$\frac{d\sigma_{pd}^{lsj}(\theta)}{d\Omega} = \sigma_{DW}^{l0l}(\theta) \tag{2.16}$$

2) Spin dependent $\sigma_{\mathbf{i}} \cdot \sigma_{\mathbf{j}}$ interaction

The interaction for this situation is given by the following form,

$$V(\vec{\mathbf{r}} - \vec{\mathbf{r}}_{\mathbf{1}})\sigma_{\mathbf{i}} \cdot \sigma_{\mathbf{j}} = 4\pi V_0 \sum_{lj} v_l(r, r_1) \sum_{\mu} \mathcal{Y}^{-\mu}_{l1j}(\hat{\mathbf{r}}) \mathcal{Y}^{\mu}_{l1j}{}^{*}(\hat{\mathbf{r}}_{\mathbf{1}}), \tag{2.17}$$

where

$$\mathcal{Y}^{\mu}_{l1j}(\hat{\mathbf{r}}) = \sum_{m} (l\, s\, m\, \mu - m | j\, \mu) i^l Y^m_l(\theta, \phi) \sigma^{\mu - m}_s, \tag{2.18}$$

and $\sigma^{m'}_s$ is the spin operator for spin s and projection m'. The matrix element for the transition amplitude then becomes

$$\begin{aligned} \big\langle J_B M_B s_b m_b | V(\mathbf{r} - \mathbf{r_1}) | J_A M_A s_a m_a \big\rangle = 4\pi V_0 \sum_{lj\mu} (J_A\, j\, M_A\, M_B - M_A | J_B\, M_B) \big\langle J_B || v_l(r, r_1) i^l \mathcal{Y}_{l1j}(\hat{\mathbf{r}}_{\mathbf{1}}) || J_A \big\rangle \\ \times (-1)^{2j} 2\sqrt{s_s(s_a+1)} (l\, s\, m\, \mu - m | s_a\, m_a) \\ \times (s_b\, s\, m_b\, m_a - m_b | s_a\, m_a)(-i)^l Y^m_l(\hat{\mathbf{r}})^* \frac{1}{\sqrt{2J_B+1}}, \end{aligned} \tag{2.19}$$

where for the usual microscopic interaction, $s = 1$ and $s_a = s_b$.

The program DWUCK4 computes the form factor,

$$f_{lsj}(r) = 2\sqrt{s_a(s_a+1)} i^l \big\langle j'_1 j'_2 J_B || v_l(r, r_1) \mathcal{Y}_{l1j}(\hat{\mathbf{r}}_1) || j_1 j_2 J_A \big\rangle \sqrt{\frac{2l+1}{(2j+1)(2J_A+1)}}. \tag{2.20}$$

In the above expression we have used the reduced matrix element [2]

$$\big\langle s_a || \sigma || s_a \big\rangle = 2\sqrt{s_a(s_a+1)}. \tag{2.21}$$

If the spin of the projectile changes such as in the (^{6}Li,^{6}He) reaction, then the matrix elements must be scaled by the factor,

$$\frac{\big\langle s_b || \sigma || s_a \big\rangle}{2\sqrt{s_a(s_a+1)}.} \tag{2.22}$$

d) Two-nucleon transfer reaction

The description of a typical transfer such as the (p,t) reaction is given in ref[6]. In this article the normalization condition gives the cross section for the reaction as,

$$\frac{d\sigma^{lsj}_{pd}(\theta)}{d\Omega} = D_0^2 \big(\frac{\pi\Delta^2}{2}\big)^{\frac{3}{2}} \big(\frac{\Delta'}{\Delta}\big)^6 (T_B\, 1\, N_B\, 1 | T_A\, N_A) \sigma^{lsj}_{DW}(\theta)/(2j+1) \tag{2.23}$$

where for $\Delta' = \Delta = 1.70$ fm, $(\pi\Delta^2/2)^{\frac{3}{2}} = 9.672$ and $(T_B\, 1\, N_B\, 1 | T_A\, N_A)$ is the isospin projection factor.

For the (t,p) reaction, we have

$$\frac{d\sigma^{lsj}_{pd}(\theta)}{d\Omega} = D_0^2 \big(\frac{\pi\Delta^2}{2}\big)^{\frac{3}{2}} \big(\frac{\Delta'}{\Delta}\big)^6 (T_B\, 1\, N_B\, 1 | T_A\, N_A) \big(\frac{2J_B+1}{2J_A+1} \sigma^{lsj}_{DW}(\theta)/(2j+1). \tag{2.24}$$

Typical values for D_0^2 are in the range $15 - 30$.

Since the two-nucleon transfer reaction is a coherent process involving the two transfered nucleons it is important to use the proper convention for the phasing of the amplitudes of the two-nucleon wave function

which arise from the overlap of the initial and final target states. This code DWUCK4 and the coupled channels code CHUCK3 assume that the spherical harmonic functions carry a time-reversal phase of i^l. This convention requires that the amplitudes will need to be multiplied by an extra phase

$$(i)^{l1+l2-LTR}$$

when they are derived from the usual nuclear structure calculations. Here, the l_i are the angular momenta for the two orbitals involved in the amplitude and LTR is the orbital angular momentum transfer. Hence, a target overlap function between two ground states which contains components from different shells for example will have the form,

$$\psi(\mathbf{r_1}, \mathbf{r_2}) = a_{1,1}(p_{1/2})^2 + a_{9,9}(g_{9/2})^2,$$

where the coefficients $a_{i,j}$ are positive and the plus sign gives the correct phasing. A standard check to make is to examine the output of the code for a two-nucleon transfer case and make sure that for a ground state to ground state transition that the contributions from each pair of orbitals at a large radius in the form factor have constructive coherence. This information is listed in the output.

3. Description of the Input for DWUCK4

The input to the program is defined by seven input blocks, several of which require more than one line of input. The first four input blocks specify the basic input to the program. The next two blocks, 5 and 6, are multi-line blocks which specify the the initial and final distorted waves and are structured so that potentials consisting of several terms may be "stacked" and accumulated. Finally, the seventh block describes the form factor and its input depends on the particular reaction model chosen. The input scheme has been designed for flexibility, hence some redundancy in the input is present and input consistency is not enforced. The formats are presented in their FORTRAN 77 form. In many computer system applications, an abbreviated form with comma separators is allowed, e.g. a valid form for input block 2 is 36. ,0. ,5.

Input Block 1 (1 line) — ICON(20), ALPHA — FORMAT(20I1, A60)

i	ICON(i)	Description
1	0	Do not read in input block 2 (use default or the angle data of the previous case.)
	1	Read input block 2 (angle data).
	9	Stop program and exit.
2	0	Collective or particle transfer nuclear model.
	2	Microscopic inelastic nuclear model.
3	0	Use the same radial form factor for each l-transfer (read in only one set of input block 7).
	1	Compute a separate radial form factor for each l-transfer (read in one input block 7 for each l-transfer).
	2	Same as ICON(3) = 1 except the cross section is the coherent sum of the amplitudes from each l-transfer.
4	0	Output each radial form factor before finite range and non-local corrections.
	1	Suppress output of the radial form factor.

	2	Suppress output of radial form factor and intermediate output in the two-particle form factors.
	3	Output form factor after calculation of the finite range and non-local correction factors.
5	0	Suppress output of the elastic scattering amplitudes for the distorted waves.
	1	Output the elastic scattering amplitude $[\exp(2i\delta_{lj}) - 1]/2i$.
6	0	Suppress output of the elastic scattering cross sections for the distorted waves.
	1	Output the elastic scattering cross sections for the distorted waves.
7	0	Suppress output of the radial matrix elements.
	1	Output the radial matrix elements as defined in eq. (1.12).
8	0	Suppress output of the inelastic scattering amplitudes.
	1	Output the inelastic scattering amplitudes as defined by eq. (1.11).
9	0	Suppress the output plotting of the inelastic cross section.
	N	Output a paper plot of the inelastic cross section as a N-decade logarithmic plot.
10	0	The kinematics for the waves are calculated non-relativistically.
	1	The kinematics for the waves are caculated relativistically.
11		Not used
12	0	Suppress output of the radial wave functions for the distorted waves.
	N	Output the radial wave functions for the distorted waves at every N^{th} radial point.
13	0	Suppress output of the inelastic cross section on a separate file.
	1	Output the inelastic cross section and angles to the file FOR007.DAT.
14		Not used.
15	0	Suppress output of $K(r)^2$, the square of the local wave number for each distorted wave.
	1	Output $K(r)^2$, the square of the local wave number for each distorted wave.
16	0	Suppress output plotting of the elastic scattering cross sections for the distorted waves.
	N	Output a paper plot of elastic scattering cross sections for the distorted waves as a N-decade logarithmic plot.
17	0	Suppress the output of the extended set of spin observables.
	2	Outputs to diskfiles FOR020.DAT and FOR021.DAT a set of extended spin observables for the inelastic channel.
Alpha		Any 60 characters (including blanks) to identify the run, beginning in column 21.

Input Block 2 (1 line)	N_ANGLES, ANGLE1, D_ANGLE	FORMAT(3F8.4)
N_ANGLES	Number of angles for calculating the cross sections.	
ANGLE1	First angle (degrees). This may be zero.	
D_ANGLE	Angle increment (degrees).	

The program has a default set of angle data for the interval of 0° to 180° in 5° intervals which is used if no angle data is read in.

Input Block 3	LMAX, NLTR, (LTR(I), I=1,NLTR), (JTR(I), I=1,NLTR)	FORMAT(18I3)

(1 line)

LMAX	Maximum partial wave for the distorted waves. The limit imposed by the storage is $400/(2s_a + 2s_b + 2) - 1$ where s_a and s_b are the spins of the initial and final projectiles.
NLTR	The maximum number of angular momentum transfers to be calculated (NLTR $\leq$ 8).
LTR(I)	The angular transfer for the I^{th} form factor.
JTR(I)	Twice the total angular momentum transfer for the I^{th} radial form factor.

Input Block 4 — DR, RMIN, RMAX, COUEX, FNRNG — FORMAT(10F8.4)
(1 line)

DR	Integration step size for the radial coordinate (in fm).
RMIN	Lower cutoff radius for the radial integrals (in fm).
RMAX	Upper cutoff for the radial integrals (in fm). If RMAX is negative the automatic resizing of RMAX in the program is overridden and the input value of |RMAX| is used. The storage allocation in the program restricts the number of radial points to INT(RMAX/DR) $\leq$ 400.
COUEX	Coulomb excitation excitation scale factor.
FNRNG	Finite range correction factor.

Input Block 5 — Initial distorted wave input block
(minimum of 2 lines)

Input line 1 — E, MP, ZP, MT, ZT, r_{0c}, AC, PNLOC, 2*FS — FORMAT(10F8.4)
(Kinematic input line)

E	Laboratory energy (in MeV) of the initial projectile (must be > 0).
MP	Projectile mass (in AMU units).
ZP	Projectile charge.
MT	Target mass (in AMU units).
ZT	Target charge.
r_{0c}	Reduced charge radius ($R_c = r_{0c}\ A^{\frac{1}{3}}$) in fm.
AC	Diffuseness of charge radius (not implemented).
PNLOC	Nonlocality parameter.
2*FS	Twice spin of the projectile.

Input lines (2,...) — OPTION, VR, r_{0R}, AR, VSOR, VI, r_{0I}, AI, VSOI, POWER — FORMAT(10F8.4)
(Potential input)

OPTION	Option number for potential shape (these options are defined below).
VR	Strength of the real potential (in MeV).
r_{0R}	Reduced radius of the real potential ($R_R = r_{0R}\ A^{\frac{1}{3}}$) (in fm).
AR	Diffuseness of the real potential (in fm).
VSOR	Spin-orbit factor for the real potential.
VI	Strength of the imaginary potential (in MeV).
r_{0I}	Reduced radius of the imaginary potential ($R_I = r_{0I}\ A^{\frac{1}{3}}$) (in fm).
AI	Diffuseness of the imaginary potential (in fm).

VSOI	Spin-orbit factor for the imaginary potential.	
POWER	Extra parameter used in some potential and form factor options.	

Any number of potential options may be used on successive lines and the resulting potential will be the superposition of the individual potentials. If OPTION is a negative number then that option will be the last potential. The program will then continue to the next input block. If the OPTION is zero then no potential will be computed and the program will process the next input block.

Input Block 6 (minimum of 2 lines)	Final distorted wave input block	
Input line 1 (kinematic input line)	QCODE, MP, ZP, MT, ZT, r_{0c}, AC, PNLOC, 2*FS	FORMAT(10F8.4)
QCODE	Q-value for the reaction (in MeV).	

The remainder of the parameters on this line are interpreted in exactly the same manner as for input block 5 but describe the kinematics and potential for the final distorted wave. The kinematic input line is then followed by the lines (2, ...) describing the potential as in input block 5.

Input Block 7 (minimum of 2 lines)	The radial form factor for the collective model and particle transfer reactions.	
Input line 1 (Kinematic input line)	E, MP, ZP, MT, ZT, r_{0c}, AC, PNLOC, 2*FS	FORMAT(10F8.4)
E	Binding energy of the single particle (in MeV).	
MP	Single particle mass (in AMU units).	
ZP	Single particle charge.	
MT	Core nucleus mass (in AMU units).	
ZT	Core nucleus charge.	
r_{0c}	Reduced charge radius ($R_c = r_{0c}\ A^{\frac{1}{3}}$) (in fm).	
AC	Diffuseness of charge radius (not implemented).	
PNLOC	Nonlocality parameter.	
2*FS	Twice the spin transfer of the form factor.	
Input lines(2, ...) (Potential lines)	OPTION, VR, r_{0R}, AR, VSOR, VI, r_{0I}, AI , VSOI, POWER	FORMAT(10F8.4)

These input lines have the same meaning as the same lines of input block 5.

If the parameter E in line 1 is zero then the potential defined by lines (2, ...) is taken to be the radial form factor. This is the standard option for the inelastic scattering collective model. If the parameter E in the first line of this block is non-zero then the program computes an eigenfunction solution using the potential defined by lines (2, ...). In this case the additional line of input below must be added to define the quantum numbers and other needed parameters.

Extra input line (Kinematic input line)	FNODE, FL, 2*FJ, 2*FS, VTRIAL, FISW	FORMAT(10F8.4)

FN	Number of nodes in the radial function excluding the origin and infinity.
FL	Orbital angular angular momentum for the radial function.
2*FJ	Twice the total angular momentum quantum number for the radial function.
2*FS	Twice the intrinsic spin of the radial function.
VTRIAL	Scaling factor for the potential defining the radial function. If VTRIAL is zero a default value of +60.0 is used.
FISW	Search control for the integration of the radial function.

= 0. Search on the strength of the potential scale factor VTRIAL keeping the binding energy E fixed.
= 1. Search on the binding energy E keeping the potential strength VTRIAL fixed.

= 2. No search (valid for E $>$ 0 only). The radial wave function is calculated for the given energy E and strength VTRIAL and matched to the asymptotic solution with the calculated phase shift.

Since the total potential is a product of VTRIAL and the forms calculated by lines (2, ...), the result must be negative in order to give a usual bound state function, i.e. VTRIAL*VR must be less than zero (for OPT = 1.0).

Input Block 7 Description of the radial form factor for the microscopic inelastic scattering model.

(Used when ICON(2) = 2)

Input line 1 CONTROL, OPCODE, FLMU, VZERO, FJ2, FJI, FJF FORMAT(10F8.4)
(Option input line)

CONTROL	= 0.0, read no input blocks for the single particle and terminate the form factor calculation. = 1.0, read one input block 7 for the single particles and use this radial function for both particles. = 2.0, read two input blocks 7, one for each single particle.
OPCODE	= 0.0, not an option. = 1.0, calculate form factor using a Yukawa potential $V(r) = V_0 \exp(-\mu r)/(\mu r)$. = 2.0, calculate form factor using a Coulomb potential. = 3.0, calculate form factor using a tensor force potential. = 4.0, not used. = 5.0, calculate a microscopic two-nucleon transfer form factor. = 6.0, calculate a zero-range knockout form factor.
FLMU	= Inverse range parameter (μ) in the inelastic scattering model or rms radius of a Gaussian wave function used in the two-nucleon transfer form factor.
VZERO	= strength of the potential used in options 1, 2, and 3 above. = amplitude of the configuration for the two-nucleon-transfer reaction of option 5. = volume integral of the two-body potential in the zero-range knockout model in option 6.
FJ2	= Twice spin of the core (j_2) to which the active single particle is coupled.
FJI	= Twice the spin of the initial nucleus $\|\vec{\mathbf{j_1}} + \vec{\mathbf{j_2}}\|$ = JI (J_A in section 2a)
FJF	= Twice the spin of the final nucleus $\|\vec{\mathbf{j_1'}} + \vec{\mathbf{j_2}}\|$ = JF (J_B in section 2a)

The values for the spins j_1 and j_1' are taken from the input line defining the quantum numbers in the input block 7 specified below.

Input lines (2, ...) Following input line 1 insert one or two input blocks (depending on the value of CONTROL in line 1) defining the single particle orbitals. The input lines are identical to those of input block 7 for the single particle transfer case. The program will add the form factors coherently until it encounters a negative or zero value for CONTROL.

Potential options available for input blocks 5, 6, and 7.

OPT = 1.0 Volume Wood-Saxon potential

$$V(r) = V_R f(x_R) + iV_I f(x_I)$$

where

$$f(x_i) = 1.0/[1 + \exp(x_i)] \quad \text{and where} \quad x_i = (r - r_{0i}\mathrm{MT}^{\frac{1}{3}})/A_i.$$

Note that in order to have an attractive real and absorptive imaginary potentials both VR and VI must be negative.

OPT = 2.0 Surface Wood-Saxon (or derivative) potential.

$$V(r) = V_R g(x_R) + iV_I g(x_I)$$

with

$$g(x) = \frac{df(x)}{dx},$$

where x is defined the same as for OPT = 1.0. This form of the potential has *no* factors of 4 in its definition so that the strength V_I is related to W_D by $V_I = 4W_D$. Further, in order that the potential have an attractive real and absorptive imaginary properties, V_R and V_I must be positive.

OPT = 3.0 Second derivative Wood-Saxon potential.

$$V(r) = V_R h(x_R) + iV_I h(x_I)$$

where

$$h(x) = \frac{d^2 f(x)}{dx^2}.$$

OPT = 4.0 Spin-orbit potential from a volume Wood-Saxon form.

$$V_{LS}(r) = \big(-V_R \frac{1}{r}\frac{df(x_R)}{dr} - iV_I \frac{1}{r}\frac{df(x_I)}{dr}\big)\mathbf{L}\cdot\mathbf{s}.$$

The above potential is defined in terms of $\mathbf{L}\cdot\mathbf{s}$ without any $(\hbar/m_\pi c)^2 \sim 2$ factor. The strength V_R is about four times the strength of the forms defined with the $(\hbar/m_\pi c)^2$ factor and an $\mathbf{L}\cdot\sigma$ operator.
The VSOR and VSOI parameters in the input line for the OPT = 1.0 case will also give a spin-orbit potential using the same geometry. The VSOR and VSOI are interpreted as a non-dimensional multiplier (λ in the so-called Thomas form [1]). The potential is written as,

$$V_{LS}(r) = \big(-V_R \frac{\text{VSOR}}{45.2}\frac{1}{r}\frac{df(x_R)}{dr} - iV_I \frac{\text{VSOI}}{45.2}\frac{1}{r}\frac{df(x_I)}{dr}\big)\mathbf{L}\cdot\mathbf{s}.$$

OPT = 5.0 Spin-orbit potential from a surface Wood-Saxon form.

$$V_{LS}(r) = \big(-V_R \frac{1}{r}\frac{dg(x_R)}{dr} - iV_I \frac{1}{r}\frac{dg(x_I)}{dr}\big)\mathbf{L}\cdot\mathbf{s}.$$

OPT = 6.0 Volume Wood-Saxon potential with a r^{POWER} factor.

$$V(r) = \big(V_R f(x_R) + iV_I f(x_I)\big)r^{\text{POWER}}$$

OPT = 7.0 Surface Wood-Saxon potential with a r^{POWER} factor.

$$V(r) = \big(V_R f(x_R) + iV_I f(x_I)\big)r^{\text{POWER}}$$

OPT = 8.0 External potential or form factor.

A potential or form factor will be read in from the input file. In addition to the potential option line, an additional line will specify the number of radial points and whether the fuction is to be added to the real or imagimary part of the potential or form factor. Then the lines containing the function will be read in. The radial increment of the function that is read in must match that of the distorted wave or form factor. In addition, a real function will be scaled by VR and an imaginary function will be scaled by VI. If the VR or VI value is zero or left blank the function will not be scaled.

Input line 1	F1, F2	FORMAT(10F8.4)
F1	Number of radial points to be read in.	
F2	= 0.0 considers the function to be real.	
	= 1.0 considers the function to be imaginary.	
input lines (2, ...)	(FF(I), I=1,F1)	FORMAT(5E16.7)

OPT = 9.0 Normalized harmonic oscillator.

$$V(r) = V_R N \mathcal{L}(r/r_{0R}) \exp(-\frac{1}{2}(r/r_{0R})^2.$$

where $\mathcal{L}(x)$ is a Laguerre polynomial and N is the normalization constant for the functions such that

$$N^2 \int_0^\infty [\mathcal{L}(r/r_{0R})]^2 \exp[-(r/r_{0R})^2] r^2 dr = 1.$$

Note that the radius parameter is the reduced radius r_{0R}.

OPT = 10.0 Gaussian r^{POWER}.

$$V(r) = V_R \exp(-(r/r_{0R})^2) r^{\text{POWER}}.$$

Again note that the radius parameter is the reduced radius r_{0R}.

OPT = 11.0 Legendre polynomial expansion of a volume Wood-Saxon potential.

$$V(r) = \int \left(V_R f[x_R(r,\theta)] + iV_I f[x_I(r,\theta)]\right) Y^0_{\text{LTR}}(\theta) d\Omega_r,$$

where

$$f(x) = \frac{1}{1+\exp(x)}$$

and

$$x_i = [r - R_i(1 + \beta_\lambda Y^0_\lambda)]/a_i.$$

Here LTR is the orbital angular transfer for this form factor, λ, the value of LAM, is the order of the multipole of the deformation and β_λ is the deformation parameter. An extra input line must follow this potential option with the values of β_λ and LAM.

Extra input line BETA, LAM FORMAT(2F8.4)

BETA Value of β_λ, the deformation parameter.

LAM Value of the deformation order λ which can be different from LTR.

OPT = 12.0 Legendre polynomial expansion of a surface Wood-Saxon potential.

$$V(r) = \int \left(V_R \frac{df[x_R(r,\theta)]}{dx_R} + iV_I \frac{df[x_I(r,\theta)]}{dx_I}\right) Y^0_{\text{LTR}}(\theta) d\Omega_r,$$

where f(x) and x have the same meanings as in option 11. In both of the options 11 and 12, the potentials for the distorted waves are calculated for LTR = 0 only, but LAM can have any value, but when these options are used as form factors the angular momentum transfer LTR is used and can take on any value.

Form factor options available for input block 7.

These options are generally used for the collective model cases, *i.e.* when ICON(2) = 0 and when the energy parameter E = 0.0 in the first line of input block 7. These options have the same basic form as in input blocks 5 and 6 above except for the following modifications which have been made for convenience in treating the collective model.

OPT = 1.0 Volume Wood-Saxon potential

$$V(r) = V_R f(x_R)(R_R/a_R)^{(\mathrm{POWER}+1.0)} + iV_I f(x_I)(R_I/a_I)^{(\mathrm{POWER}+1.0)}.$$

The default value for POWER = 0.0 gives the usual factor of (R/a) in the definition of the collective model.

OPT = 2.0, 3.0 As in the case for OPT = 1.0 the real and imaginary components of the form factor are multiplied by their appropriate $(R/a)^{(\mathrm{POWER}+1.0)}$ factor.

OPT = 4.0, 5.0 These options are not available as form factors.

OPT = 6.0-12.0 These options give the same functional form and normalizations as in the same numbered options in input blocks 5 and 6.

4. Sample Cases—Input and Output

Two cases of sample input are shown below. The first example describes excitation of a 3- excited state of ^{56}Fe by the inelastic scattering of 22.5 MeV protons using a collective model where the form factor is the first derivative of the optical potential. Here the Coulomb excitation contribution is included. The second case gives the input for a ^{40}Ca(d,p)^{41}Ca reaction where the model assumes the stripping of a $1f_{7/2}$ neutron. The calculation includes a finite range correction with a radius parameter of 0.70 fm. In both cases the optical potentials for the projectiles are composed of volume-real and surface-imaginary potentials and also include spin-orbit potentials. The final input line (beginning with a 9) signals the end of data.

```
1000000030000000     Fe56(p,p')Fe56* @22.5 MeV to 4.51 MeV 3- state, Coul. exc.
+37.       +00.      +05.
+15+01+03
+00.10     +00.      +15.      +01.00
+22.5      +01.0078  +01.      +56.      +26.      +01.25    +00.      +00.      +01.
+04.       -28.2     +01.25    +00.735   +00.      +00.      +01.25    +00.735   +00.
+01.       -46.38    +01.25    +00.735   +00.      +00.      +01.25    +00.735   +00.
-02.       +00.      +01.25    +00.437   +00.      +61.4     +01.25    +00.437   +00.
-04.51     +01.0078  +01.      +56.      +26.      +01.25    +00.      +00.      +01.
+04.       -28.2     +01.25    +00.735   +00.      +00.      +01.25    +00.735   +00.
+01.       -46.38    +01.25    +00.735   +00.      +00.      +01.25    +00.735   +00.
-02.       +00.      +01.25    +00.437   +00.      +61.4     +01.25    +00.437   +00.
+00.       +01.0078  +01.      +56.      +26.      +01.25    +00.      +00.      +00.
+02.       -46.38    +01.25    +00.735   +00.      +00.      +01.25    +00.735   +00.
-03.       +00.      +01.25    +00.437   +00.      +61.4     +01.25    +00.437   +00.
1000000030000000     Ca40(d,p)Ca41 @13.7 MeV to 7/2- g.s. local, finite range
+37.       +00.      +05.
+15+01+03+07
+00.10     +00.      +12.      +00.      +00.70
+13.70     +02.      +01.      +40.      +20.      +01.40                        +02.
+01.       -97.40    +01.112   +00.875                       +01.562   +00.477
-02.                 +01.112   +00.875             +70.00    +01.562   +00.477
```

```
+06.141  +01.   +01.   +41.   +20.   +01.25                    +01.
+01.     -49.47 +01.18 +00.70 +24.2          +01.252 +00.750
-02.            +01.18 +00.70          +19.80 +01.252 +00.750
-08.364  +01.   +00.   +40.   +20.          +01.25           +01.
-01.     -01.   +01.18 +00.70 +25.
+00.     +03.   +07.   +01.   +58.
9                              End of data
```

The first page of the output echoes the input blocks 1–6 and displays the derived kinematical quantities for the distorted waves such as the center-of-mass energies, wave numbers, Coulomb parameters, etc. The second page outputs the same information for input block 7 including output of the radial form factor. The third page gives output for the inelastic cross sections and spin observables in the first six columns; the next six columns give the same output appropriate to the time-reversed reaction. The final two pages of output are paper plots of the cross section and asymmetry parameter which is the polarization for the time-reversed reaction.

Typical numbers to check in the output are the the absorption cross section REACSIG for the distorted waves and the total inelastic cross section Tot-Sig. The correct value for REACSIG confirms that the elastic scattering has been calculated correctly. The correct value of Tot-Sig indicates that the reaction theory part of the program has also been entered and calculated correctly. Values of the cross sections for these two cases are given in the table below in fm^2.

Value	^{56}Fe(p,p$'$)^{56}Fe	^{40}Ca(d,p)^{41}Ca
REACSIG 1	1.1276E+02	1.3394E+02
REACSIG 2	1.0931E+02	9.9707E+01
Tot-Sig	2.9046E+01	1.8393E+00

5. Things to Do

The user should first run the two sample cases above and verify that the output numbers check with those in the table. Following this it might be instructive to change the (d,p) input case to other values of transferred angular momenta (ℓ, j)— say $p_{3/2}$ and $p_{1/2}$— which will illustrate the strong angular-distribution dependence on ℓ and weak dependence on j. Similar runs can also be made for the (p,p$'$) case. Finally for the analytically inclined, it is possible to obtain a closed-form solution for a plane-wave $\ell = 0$ case with a harmonic-oscillator form factor (option 9 above). This check can then be exploited by the user to understand in detail the normalization factors employed in the DWUCK4 program.

References

[1] G. R. Satchler, *Direct Nuclear Reactions*, Oxford University Press, Oxford OX2 6DP (1983).

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[3] Aage Bohr and Ben R. Mottelson, *Nuclear Structure, Vol I and II*, W. A. Benjamin, New York (1969).

[4] H. H. Barschall and W. Haeberli, *Polarization phenomena in nuclear reactions*, University of Wisconsin Press, Madison (1969).
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[6] H. Baer *et al.*, Ann. Phys. (NY) **76**, (1973) 437.

APPENDIX

The table below shows some typical values for the normalization of various pick-up and stripping reactions. Some of these values are derived from fitting reaction data and others are derived theoretically from simple models for the light projectiles. The only values which are on a firm basis are for the (d,p) and (p,d) reactions. Use of the others should be viewed with caution as it is important to discriminate between the D_0^2 values which are used with the finite range correction and those which are used without the correction. This is because the finite range correction is large for the reactions other than the (d,p) reaction and the renormalization of the value for D_0^2 to approximately take into account the correction is significant and will affect the values of the derived spectroscopic factors by by 20% to 100%.

Reaction	D_0^2	FNRNG parameter
(d,p)	1.55	0.621
(^{3}He,d)	4.42	0.770
(^{3}H,d)	5.06	0.845
(^{4}He,^{3}He)	(24-46)	0.7
(^{4}He,^{3}H)	(24-46)	0.7
Reaction	$D_0^2(2s_a+1)/(2s_b+1)$	FNRNG parameter
(d,p)	2.33	0.621
(d,^{3}He)	2.95	0.770
(d,^{3}H)	3.33	0.845
(^{3}He,^{4}He)	(12-23)	0.7
(^{3}H,^{4}He)	(12-23)	0.7

Table of reaction normalization coefficents.

The source files for this version of DWUCK4 contain a number of routines in the files DW4VAX.FOR, DW4PC.FOR, DW4IBM.F and DW4UNIX.FOR. These files will interface the program to the time, date and disk file opening routines in the three systems, DEC's VMS, Microsoft's DOS, IBM RS6000 Unix system or the Berkeley UNIX systems. If the user's computer system differs from any of these four, then the user will have to code similar routines in order to have the input, output and scratch files opened properly and the output labelled with the time and date.